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# Ferrohydrodynamic pumping in spatially traveling sinusoidally time-varying magnetic fields

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## Abstract

In this paper, we present a numerical method to simulate the dynamics of ferrofluids in spatially traveling, sinusoidally time-varying magnetic fields. Actuation using traveling waves involves magnetic body forces and torque on the magnetic moment of nanoparticles, both of which affect the overall pumping.

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## 1. Introduction

Ferrofluids are stable colloidal suspensions of nano-size ferromagnetic particles in either an aqueous or an oil-based solvent [1]. In the presence of an oscillating magnetic field gradient within a ferrofluid, magnetic forces and torque are developed to drive magnetic particles, which subsequently draw along the liquid solvent carrier. This allows continuous actuation and precise positioning of the ferrofluid using a magnetic field. Ferrofluids have found their way into a variety of applications, such as sealing, damping and blood separation.

Ferrohydrodynamic pumping in spatially uniform, sinusoidally time-varying magnetic fields has been studied extensively in the past [2]. Here we examine a different case where the applied sinusoidal field is a spatially traveling wave.

## 2. Governing equations

The weak field approximation of the magnetization relaxation equation for a ferrofluid with simultaneous magnetization, convection with position-dependent velocity  $\mathbf{v}$ , and reorientation due to particle spin  $\boldsymbol{\omega}$  is [1]

$$\frac{\partial \mathbf{M}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{M} - \boldsymbol{\omega} \times \mathbf{M} + \frac{1}{\tau} [\mathbf{M} - \chi_0 \mathbf{H}] = 0, \quad (1)$$

where  $\tau$  is a relaxation time constant and  $\chi_0$  is the effective magnetic susceptibility. Consider a planar ferrofluid layer (pumping in cylindrical case has been reported in Ref. [3]) as shown in Fig. 1, where the flow velocity has only a  $z$ -directed component and the spin velocity has only a  $y$ -directed component. Both velocities will change with the  $x$  coordinate, i.e.,

$$\mathbf{v} = v_z(x) \mathbf{i}_z, \quad \boldsymbol{\omega} = \omega_y(x) \mathbf{i}_y. \quad (2)$$

The coupled linear and angular momentum conservation equations for an incompressible fluid experiencing a body force density  $\mathbf{f}$  and a torque density  $\mathbf{T}$  are

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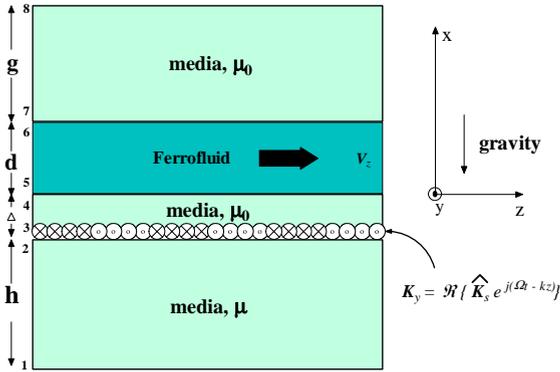


Fig. 1. A ferrofluid segment inside a planar channel is magnetically stressed by a spatially traveling sinusoidal current sheet.

given by [2]

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \mathbf{f} + 2\zeta \nabla \times \boldsymbol{\omega} + (\zeta + \eta) \nabla^2 \mathbf{v} - \rho g \mathbf{i}_x, \quad (3)$$

$$I \left[ \frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{v} \cdot \nabla) \boldsymbol{\omega} \right] = \mathbf{T} + 2\zeta (\nabla \times \mathbf{v} - 2\boldsymbol{\omega}) + \eta' \nabla^2 \boldsymbol{\omega}, \quad (4)$$

where  $\rho$  is the mass density,  $g$  is the gravitational acceleration,  $p$  is the pressure,  $\zeta$  is the vortex viscosity,  $\eta$  is dynamic viscosity,  $I$  is the moment of inertia density, and  $\eta'$  is the shear coefficient of spin viscosity. For  $0 < x < d$ , i.e., inside the ferrofluid, the magnetic force density is

$$\mathbf{f} = \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} \quad (5)$$

and the torque density is given by

$$\mathbf{T} = \mu_0 (\mathbf{M} \times \mathbf{H}). \quad (6)$$

### 3. Boundary conditions

In a real physical system, the spin viscosity  $\eta'$  is typically very small but nonzero, which allows independent boundary conditions (nonslip in this case for both  $\mathbf{v}$  and  $\boldsymbol{\omega}$ ) to be imposed. For completeness, we also study the case when the spin viscosity  $\eta' = 0$ , which reduces the governing flow Eqs. (3) and (4) to a second-order system. The boundary conditions for  $\boldsymbol{\omega}$  will then have to be dropped. In either case (of a zero or finite spin viscosity), determining the magnetic field throughout the ferrofluid requires specifying the boundary conditions for Eq. (1); this can be accomplished by solving Maxwell's Equations for each material layer outside the ferrofluid, resulting in transfer relations for each field Fourier component of the field [4]. Just below and above the current sheet in Fig. 1, for instance, these relations

look like

$$\begin{pmatrix} \widehat{B}_{1,x} \\ \widehat{B}_{2,x} \end{pmatrix} = j\mu \begin{pmatrix} -\coth(kh) & \frac{1}{\sinh(kh)} \\ -\frac{1}{\sinh(kh)} & \coth(kh) \end{pmatrix} \begin{pmatrix} \widehat{H}_{3,z} \\ \widehat{H}_{4,z} \end{pmatrix}, \quad (7)$$

$$\begin{pmatrix} \widehat{B}_{3,x} \\ \widehat{B}_{4,x} \end{pmatrix} = j\mu_0 \begin{pmatrix} -\coth(k\Delta) & \frac{1}{\sinh(k\Delta)} \\ -\frac{1}{\sinh(k\Delta)} & \coth(k\Delta) \end{pmatrix} \begin{pmatrix} \widehat{H}_{3,z} \\ \widehat{H}_{4,z} \end{pmatrix}, \quad (8)$$

where  $\Delta$  and  $h$  are the layers thickness,  $\mu_0$  and  $\mu$  are the permeabilities,  $j$  is  $\sqrt{-1}$ . Note that Eqs. (7) and (8) are valid for material layers *outside* the ferrofluid only. Once they are solved for each outer layer and coupled with the ferrohydrodynamics of the fluid layer, the complex field amplitude at each boundary surface can be

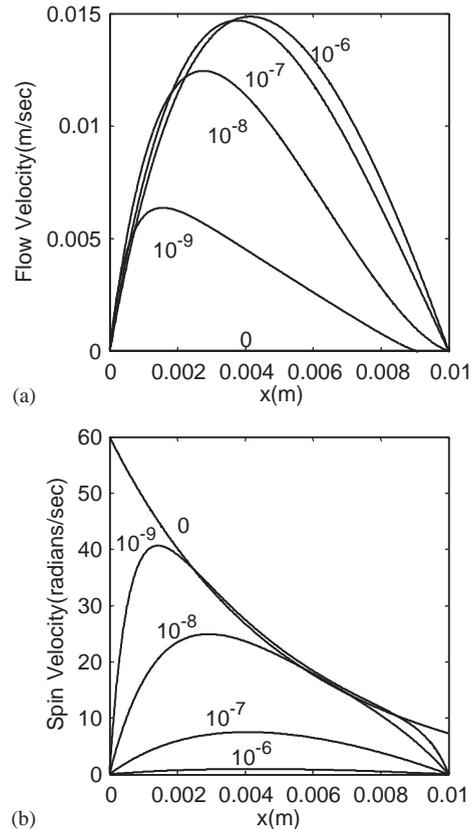


Fig. 2. (a) Flow velocity and (b) spin velocity for various spin viscosity  $\eta'$  values.  $\mu = 3\mu_0$ ,  $K_s = 1000$  A/m, excitation frequency = 50 kHz, height of channel  $d = 10$  mm, traveling wave period  $\lambda = 2\pi/k = 62.8$  mm,  $\chi_0 = 1.7$ ,  $\tau = 10 \mu\text{s}$ ,  $\eta = 0.006$  kg/m s,  $\zeta = 0.0008$  kg/m s,  $\eta' = 0, 10^{-9}, 10^{-8}, 10^{-7}, 10^{-6}$  kg/m s. The external pressure gradient is zero, the wall thickness is negligible. Pumping is very small but finite when  $\eta' = 0$ .

obtained as a function of the amplitude,  $K_s$ , of the traveling wave current sheet at the outer wall (Fig. 1).

**4. Numerical method**

First, initial estimates of  $\mathbf{v}$  and  $\boldsymbol{\omega}$  are made to calculate the magnetic field inside the ferrofluid by using Eq. (1). Force density  $\mathbf{f}$  and torque density  $\mathbf{T}$  are then determined from Eqs. (5) and (6). Later, new sets of  $\mathbf{v}'$  and  $\boldsymbol{\omega}'$  are obtained by solving Eqs. (3) and (4) (in the steady-state, negligible inertia limit) numerically via a second-order central-differencing scheme. The process of solving Eqs. (1)–(6) is iterated until the normalized changes in  $\mathbf{v}'$  and  $\boldsymbol{\omega}'$  fall below a preset tolerance.

**5. Numerical results and discussion**

The ferrofluid flow velocity and magnetic particle spin profiles for a planar fluidic channel are shown in Fig. 2, where the parameters are chosen to correspond to those of a commercially available, oil-based ferrofluid. Traveling wave magnetic fields develop both magnetic body force and torque pumping. Fig. 3 shows the pumping for various traveling wave periods. Average flow velocities for various traveling wave periods and magnetic field frequencies are shown in Fig. 4. The flow velocity is strongly dependent on the traveling wave period. Maximum flow velocity is achieved when the product of the excitation wave number and the height of the ferrofluid channel approaches unity. Once geometric dimensions are chosen, the flow velocity can be precisely

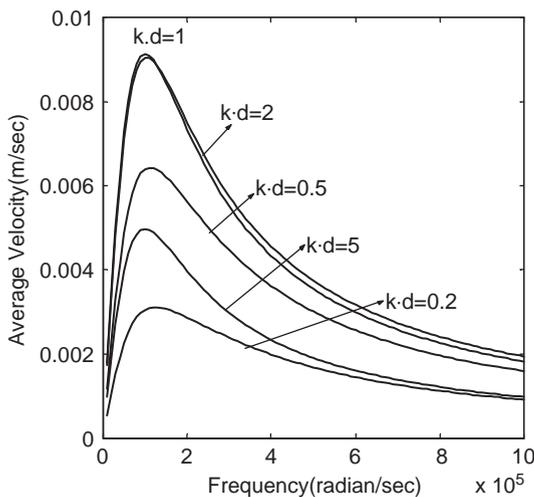
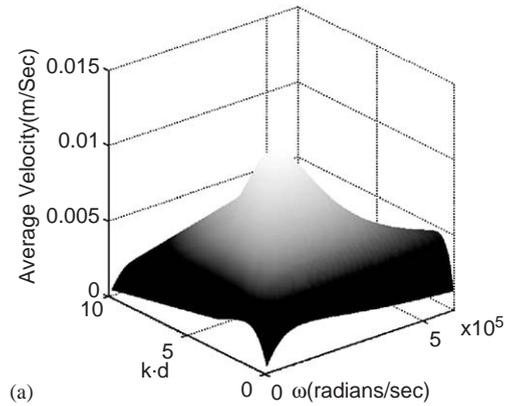
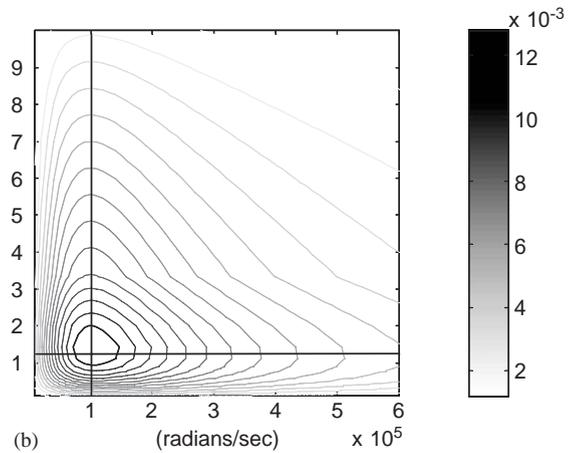


Fig. 3. Pumping for various traveling wave periods.  $\mu = 3\mu_0$ ,  $K_s = 1000$  A/m, height of channel  $d = 10$  mm, wave number  $k = 2\pi/\lambda$ ,  $\eta' = 10^{-8}$  kg/m.s. Other properties are the same as those in Fig. 2.



(a)



(b)

Fig. 4. (a) 3-D and (b) contour plot of average flow velocity of ferrofluid versus the product of wave number and the height of the channel ( $k \cdot d$ ) for various applied magnetic field frequencies. Maximum pumping is achieved when the product is approximately unity. Pumping efficiency increases as the applied magnetic field frequency becomes comparable to the reciprocal of the relaxation time constant of magnetic particle in the ferrofluid. Same material properties and operating conditions as in Fig. 3 are assumed here. (a) Average velocity versus magnetic field frequency and  $k \cdot d$  (b) Contour plot of average velocity versus magnetic field frequency and  $k \cdot d$ .

controlled by the applied magnetic field frequency. As shown in Fig. 4, maximum flow velocity is achieved when the applied magnetic field frequency is close to the reciprocal of the relaxation time constant of magnetic particles.

**6. Conclusion**

In this work, a numerical method to solve the steady-state ferrofluid dynamics in spatially traveling, sinusoidally time-varying magnetic fields has been developed. It is found that maximum pumping occurs

when the excitation wave number is the inverse of the ferrofluid channel thickness.

It is interesting to note that ferrohydrodynamic pumping with traveling waves bears a resemblance to induction motors, where a stator creates a traveling excitation that pulls the rotor along, and where there exists an optimum slip frequency that is determined by the  $L/R$  time constant of the rotor. In the case of ferrohydrodynamic pumping with travelling waves, this optimum frequency is determined by the time constant that dictates the alignment of particle magnetization with the applied magnetic field.

### **Acknowledgement**

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